Remark to "On the Description of Fermion Systems in Boson Representations"

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The formulae of boson powers commutation relation used by B. Sorensen [Nucl. Phys. A **119** (No 1) (1968), 65] in his calculations is erroneous. We provide the correct formulae.

In the paper of B. Sorensen "On the Description of Fermion Systems in Boson Representations" [1] one can find for bosons powers commutation relations the formula (2.5 page 66)

$$\left[a^n,(a^\dagger)^m\right]=\begin{cases}\sum_{l=0}^{n-1}\frac{(l+1)m!}{(m-n+l)!}\,(a^\dagger)^{\,m-n+l}a^l & \text{for}\quad n\le m\\ \\ \sum_{l=0}^{m-1}\frac{(l+1)n!}{(n-m+l)!}\,(a^\dagger)^l a^{n-m+l} & \text{for}\quad n\ge m\end{cases}\tag{1}$$

where a and $a^\dagger$ are the annihilation and creation boson operators.

Dealing with this formula we observed that it is erroneous. Let us for example compare the straitforwardly calculated results

$$\left[a^4,(a^\dagger)^4\right]=24+96a^\dagger a+72a^{\dagger 2}a^2+16a^{\dagger 3}a^3\tag{2}$$

with the corresponding results that gives formula (1) for $n=m=4$:

$$\left[a^4,(a^\dagger)^4\right]=24+48a^\dagger a+36a^{\dagger 2}a^2+16a^{\dagger 3}a^3;\tag{3}$$

In this remark, starting with the identity:

$$[A, B^n] = \sum_{k=0}^{n-1} B^k \, [A, B] \, B^{n-k-1} \tag{4}$$

and putting the operators $A \longrightarrow a$; $B \longrightarrow a^\dagger$ and then using the induction method, or directly applying the Leibnitz relations :

$$\begin{aligned} &\tfrac{d^n}{dx^n}(x^n \ f) = \sum_{l=0}^{n} \tbinom{n}{l} \left(\tfrac{d^{n-l}}{dx^{n-l}} x^n\right) \left(\tfrac{d^l}{dx^l} f\right) \\ &\text{with} \quad a \longrightarrow \tfrac{d}{dx} \quad \text{and} \quad a^\dagger \longrightarrow x \end{aligned} \tag{5}$$

we had derived the right expressions for the boson powers commutation relation:

$$\left[a^n, (a^\dagger)^m\right] = \begin{cases} \sum_{l=0}^{n-1} \frac{m!}{(m-n+l)!} \binom{n}{l} (a^\dagger)^{m-n+l} a^l & \text{for } n \leqslant m \\ \\ \sum_{l=0}^{m-1} \frac{n!}{(n-m+l)!} \binom{m}{l} (a^\dagger)^l a^{n-m+l} & \text{for } n \geqslant m \end{cases} \tag{6}$$

Where $\binom{n}{l}$ are the Binomial coefficients.

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[1] B. Sorensen Nucl. Phys. A **119** (No 1) (1968), 65